Specification and Verification of Radiation Therapy System with Respiratory Compensation using Uppaal

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Abstract—The goal of radiation therapy is to give as much dose as possible to the target volume of tissue and avoid giving any dose to a healthy tissue. Advances of the digital control allow performing accurate plans and treatments. Unfortunately, motion compensation during the treatment remains a considerable problem. Currently, a combination of the different techniques, such as gating (restricting movement of patient) and periodic emission are used to avoid damaging healthy tissue. This paper focuses on systems that completely compensate respiratory movement (up to certain limit) and start by investigating adequacy of the existing hardware and software platform.

In this paper a radiation therapy system consisting of a HexaPOD couch with 6-degrees movement, a tracking camera, a marker (markers) and a controller is modeled. A formal un-timed model was evaluated and found to be insufficient to completely determine adequacy of the system to compensate respiratory motion. Therefore, un-timed model was extended to include time and investigated. It provides more information than un-timed model, but does not answer all interesting question. Therefore, based on the results further research directions are sketched.

Index Terms—simulation, verification, formal methods, radiation treatment, quality assurance

I. INTRODUCTION

The goals of the radiation therapy is to give as much dose as possible to the target volume of tissue and avoid giving any dose to a normal tissue. Advances of the computer-based control allow planning and performing accurate plans and treatments, however motion compensation during treatment remains a considerable problem. Different techniques to cope with such problem are analyzed in [1]. Usage of gating combined with external surrogates is overviewed in [2]. However, most of the research models and try to predict movement of the tumor, e.g. [3]–[5]. This paper, on the other hand, is interested in modeling hardware and software, which is supposed to conform to the requirements, i.e. process images and move precisely and fast. Formal methods are used for such analysis, because they provide means for rigorous modeling and analysis of diverse systems. The main reasons of the formal methods’ popularity are the following.

- **Unambiguous models.** Formal modeling languages allow defining systems unambiguously, because syntax and semantics are defined formally, and those languages include means to define non deterministic and stochastic behavior precisely, too. Moreover, for the same reasons, unambiguous refinement and code generation techniques can be applied.

- **Strict analysis techniques.** Because models are defined using languages with strict semantics, rigorous reasoning about models is possible. E.g., model checking, theorem proving and specifically designed algorithms can be used.

Quite a few techniques and tools were defined over the years, e.g. process algebras [6]–[11], timed automaton [12], hybrid automaton [13], SPIN [14] and Uppaal [15] tools and a lot more, see [16], [17] for a wider overview. Successful application of formal techniques is reported in different areas, e.g. automotive industry [18], electronics [19], industrial devices control [20] and other.

This paper investigates applicability of timed automaton [12] and Uppaal tool [15] for the design and functional analysis of a radiation therapy system consisting of a HexaPOD couch with 6-degrees movement, a tracking camera, a marker (markers) and a controller. Uppaal is an integrated tool environment for modeling, validation and verification of real-time systems modeled as networks of timed automata, extended with data types and other convenient constructions [15]. In [21] an un-timed version of the model was presented. However, the model is to abstract to determine adequacy of the system for a respiratory motion compensation task. Therefore, it was extended to include some timing properties and analyze some functional properties in [21]. In this paper timed model and timing aspects are presented in detail. Moreover, functional properties, i.e. absence of deadlocks, liveness and safety, are analyzed.

In Section II a detailed description of the radiation treatment system is provided. Then Uppaal and timed automaton in Section III are concisely introduced. In Section IV a Uppaal model of the radiation treatment system is presented, some of its properties are checked, and its applicability to further analysis is discussed. Future plans and conclusions are discussed in Section V.
II. RADIATION TREATMENT SYSTEM

Radiation treatment system under analysis\(^1\), depicted in Fig. 1, consists of the following components:

**Patient Setup Couch** is used to position the patient for the treatment, in our case the HexaPOD couch [22], [23].

**External Radiation Beam Source**, usually produced by a *medical linear accelerator*, in short, linac. In the current stage of our study it is not important, because behavior of the couch, the tracking device and the controller are analyzed.

**Tracking Device** provides information about the position of the patient. Different means and techniques can be used to perform it, see [1] for the details. A system with a stereo camera is modeled. In this paper hardcoded trajectories are used instead of dynamic input, and therefore, it is omitted.

**Controller** is a system, that controls the treatment process, in our case the controller uses information provided by the treatment plan and the HexaPOD response to control it.

A. **Experiments with HexaPOD Couch**

Technical documentation of the HexaPOD device does not provide detailed documentation of its behavior when it is used continuously, not just to move a patient into a specified position. Usually, when a new position is provided, it starts to move towards it accelerating with \(5.5\text{m/s}^2\) acceleration until it reaches \(7.6\text{m/s}\) (instead of the stated \(8\text{mm/s}\) velocity. Then, when \(5\text{mm}\) are left to the target, it starts decelerating with \(5.5\text{m/s}^2\). In case, when distance to the target is less than \(5\text{mm}\), HexaPOD accelerates until the middle of the interval and then decelerates until it reaches target and stops.

Based on these experiments and expected breath movement timing properties of the model and testing trajectories can be defined.

\(^1\)There is a diversity of radiation treatment systems, see [1] for overview of the systems relevant to this study. However, here we define just a selected setup.

III. TIMED AUTOMATON AND UPPAAL

Timed automaton [12] is one of the most popular techniques for modeling and analysis of the real-time systems. A version of automata used in Uppaal [15] is presented. Uppaal is an integrated tool environment for the modeling, simulation and verification of (complex) real-time systems. It is well-suited for systems that can be modeled as a collection of non-deterministic processes with finite control structure and real-valued clocks, communicating through channels or shared variables.

**Definition 1.** Let \(C = \{x, y, z, \ldots \}\) be a set of clocks and \(B(C)\) is the set of clock restrictions of the form \(g, g_1, g_2 := x \gg y \gg c|\geq g_1 \land g_2\) with \(x, y \in C, c \in \mathbb{N}\) and \(\gg \in \{\ll, =\ll, >\ll, \geq\}\).

**Definition 2.** A timed automaton is called as a finite directed graph \(A = (L, l_0, A, E, I)\) over \(C\) and \(B(C)\), where

- \(L\) is a finite set of locations;
- \(l_0 \in L\) is the initial location;
- \(A\) is a finite set of action names;
- \(E \subseteq L \times B(C) \times A \times 2^C \times L\) is a finite set of edges, and
- \(I : L \rightarrow B(C)\) assigns invariants to locations.

The transition \(l \xrightarrow{a,r} l'\) is written instead of \((l, g, a, r, l) \in E\). \(l\) is called the *source location* of the state, \(g\) is the guard, \(a\) is the action, \(r\) is the set of clocks to be reset and \(l'\) is the *target location*.

Timed automata can be represented as in Fig. 2. Locations are depicted as nodes of the graph, and the initial location is usually marked with a double circle. Transitions are depicted by arrows.

**Definition 3.** Let \(A = (L, l_0, A, E, I)\) be a timed automaton over a set of clocks \(C\). The timed transition system \(T(A)\) generated by \(A\) is defined as \(T(A) = (S, \text{Act}, \rightarrow_{T})\), where:

- \(S = L \times (C \rightarrow \mathbb{R}_{\geq 0})\) is a set of states \((l, v)\), where \(l\) is a location of the timed automaton and \(v\) is a clock *valuation* that satisfies the invariant of \(l\);
- \(\text{Act} = A \cup \mathbb{R}_{\geq 0}\) is the set of labels;
- two types of transitions are defined:
  - *action transitions* \((l, v) \xrightarrow{a,r} (l', v')\) such that exists an edge \((l, g, a, r, l') \in E\) where \(v\) satisfies \(g\), \(v'\) satisfies \(v'[r]\) and \(v'\) satisfies \(I(l')\),
- delay transitions \((l, v) \xrightarrow{d} (l', v')\) if \(\forall d' \in [0, d] \Rightarrow v + d' satisifies I(l)\).

Let \(v_0\) denotes the valuation such that \(v_0(x) = 0, \forall x \in C\). If \(v_0\) satisfies the invariant of the initial location \(l_0\), \((l_0, v_0)\) is called the initial state of \(T(A)\).

Timed automata are composed into a network of timed automata consisting of \(n\) timed automata \(A_i = (L_i, l_i^0, A, E_i, I_i), i = 1...n\) over a set of clocks \(C\). Let \(l = (l_1, ..., l_n)\) be a location of the network, then invariants are composed using conjunction \(I(l) = \bigwedge_{i=0}^n I_i(l_i)\).

**Definition 4.** Let \(A_i = (L_i, l_i^0, A, C, E_i, I_i), i = 1...n\) be a network of \(n\) timed automata. Let \(l_0 = (l_1^0, ..., l_n^0)\) be the initial location vector. Then the semantics is defined as a transition system \((S, s_0, \rightarrow)\), where \(S = (L_1 \times \cdots \times L_n) \times \mathbb{R}^C\) is the set of states, \(s_0 = (l_0, v_0)\) is the initial state and transition relation contains three types of transitions:

- **time flow transitions** \((l, v) \xrightarrow{d} (l, v + d)\), if \(\forall d' \in [0, d]\) holds \(v + d' \models I_\forall(l)\);
- **discrete transitions**
  - **synchronized** \(\left(\{l_1, l_2, ..., l_n\}, v\right) \xrightarrow{\tau} \left(\{l_1', l_2', ..., l_n'\}, v'\right)\) if \(\exists i \neq j\), \(\exists l_1, l_2, ..., l_n\), \(v' = I_\forall(l)\) and \(v' = I_\forall(l)\);
  - **asynchronous** \(\left(\{l_1, l_2, ..., l_n\}, v\right) \xrightarrow{\tau} \left(\{l_1', l_2', ..., l_n'\}, v'\right)\) if \(\exists l_1, l_2, ..., l_n\), \(v' = I_\forall(l)\) and \(v' = I_\forall(l)\);

There are many tools for designing real-time systems based on the theory of timed automata. For example, KRONOS performs model-checking of TCTL formulas with respect to timed safety automata [24]. The Hybrid Technology tool (HYTECH) is for analysis of embedded systems. It computes the condition under which a linear hybrid system satisfies a temporal requirement. Since times automata are particular hybrid systems they can be verified with this tool [25]. State Graph Manipulator tool (SGM) is for real-time system specification and verification. It uses various sophisticated verification techniques developed in the previous years [26].

The model-checker Uppaal is based on the theory of timed automata as well, however its modeling language offers additional features such as bounded integer variables and urgency. The query language of Uppaal, used to specify properties to be checked, is a subset of Real Time CTL (computation tree logic) [12], [15], [27]:

- \(A[]\) property invariant, property always holds in all paths;
- \(A<>\) property eventually, property holds in all paths at some moment;
- \(E<>\) property possibly, property eventually holds at some state, at least in one path;
- \(E[]\) property potentially always, property eventually holds from some state, at least in one path;
- \(p \rightarrow q\) leads to, whenever \(p\) holds eventually \(q\) will hold;
- deadlock true, if deadlock state is reachable;
- \(F\) state certain properties hold in the selected state.

IV. UPPAAL MODEL OF THE RADIATION TREATMENT SYSTEM

A work in progress is presented, a simplified version of the radiation treatment system defined in sect. II. Model presented in [21] with timing aspects is extended. Uppaal model consists of the following components:

- **Controller** that, based on its state, a treatment plan and the input from the tracking system, i.e. stereo camera, controls movement of the HexaPOD;
- **HexaPOD Buffer** that models asynchronous communication and latency between the controller and the HexaPOD.
- **Tracker**, in this case an abstraction of a tracking device (e.g., stereo camera), observes tracker placed on the HexaPOD (or patient), calculates position of the tracker, and provides it to the controller. In this model we use predefined inputs and ignores it.

V. GLOBAL DEFINITIONS, VARIABLES AND DESCRIPTION OF COMPLETE SYSTEM

Global definitions and variables are used all over the model.

We provide them below.

```c
typedef struct {
    int x;  
    int y;  
    int z;  
} POSITION;

can move_to; // Controller->HexaPODBuffer
POSITION set_target_pos = (0, 0, 0);  
urgent chan get_move; // Buffer->HexaPOD
```

System definition just instantiates all templates and merges them into a complete model.

```c
Controller = Controller_();
HexaPOD = HexaPOD_();
HexaPODBufferLat = HexaPODBufferLat_();
```

A. HexaPOD

An Uppaal model of the HexaPOD is depicted in Fig. 3. It is modeled as a one point-device with a discrete movement in three \(-x, y, z\) directions. We abstract from the acceleration and rotation. Instead of continuous behavior discrete steps on the grid with constant velocity are defined. It allows investigating an impact of the latency and the general design of HexaPOD control. The automaton consists of three locations:
**Idle:** HexaPOD waits for a command `move_to`. With this action it receives a target, and changes to **Move** location.

**Move:** HexaPOD stepwise moves towards the target, taking steps in the predefined direction of the predefined length at a constant speed. After each step it checks for a new target, and updates the current one, if necessary. When the target is reached, it changes to **TargetReached** location.

**TargetReached:** is a committed location (a special type of location, which should be left at the next step), which is used for diagnostic reasons, see sect. III.

Description of the model is defined as follows.

```
clock t;
POSITION target_pos = {0, 0, 0};
POSITION current_pos = {0, 0, 0};
void step() // make step
{
    if (current_pos.x < target_pos.x)
        current_pos.x++;
    else if (current_pos.x > target_pos.x)
        current_pos.x--;
    if (current_pos.y < target_pos.y)
        current_pos.y++;
    else if (current_pos.y > target_pos.y)
        current_pos.y--;
    if (current_pos.z < target_pos.z)
        current_pos.z++;
    else if (current_pos.z > target_pos.z)
        current_pos.z--;
}
```

**B. HexaPODBuffer**

HexaPODBuffer, depicted in Fig. 4, models asynchronous communication and latency. It consists of the following three locations.

- **Empty** location denotes an empty buffer, it awaits for an input from the Controller, i.e. the `move_to` command, and the target, and then changes to **Latency** location.

- **Latency** location is used to model delays in the system, i.e. after receiving the new target the buffer delays for a while before making it available to the HexaPOD. However, the new target can be provided to the buffer anytime.

- **Ready:** when the buffer is ready, the target can be acquired by the HexaPOD using `get_move` command (action), and location is changed to **Empty**.

Description of the buffer (just clock) is provided below.

```
clock t;
```

**C. Controller**

In the current model controller provides control commands to the HexaPOD. It consists of three locations:

- **Start** - start of the treatment program (plan).
- **Move** - the control program is in progress, control inputs provided by an array are sent to the HexaPOD at the predefined time moment.
- **Finished** denotes that the control program was completed successfully.

```
const int STEPS = 5;
int step = 0;
clock t;
typedef struct {
    POSITION pos; // position
    int tsmap; // timestamp
} PATH;

// Test trajectory
const PATH path[STEPS] = {
    {{ 2, 3, 4 }, 0},
    {{ 3, 3, 4 }, 30},
    ...,
    {{ 3, 3, 3 }, 10},
    {{ 4, 3, 3 }, 20},
    {{ 3, 3, 3 }, 20}
};
```

Fig. 3. HexaPOD in Uppaal.

Fig. 4. HexaPODBuffer in Uppaal.

Fig. 5. Controller in Uppaal.
D. Simulation and Analysis

Stepwise timed simulations allows to acquire an insight of the model behavior. However, Uppaal allows more, i.e. the conformance of the system to the selected properties can be verified. The following properties are used to analyze the model:

1) \( E<> \) Controller.Finished
   property allows to check, if there exists a path that allows for the Controller to reach its final location. This property holds for the model under analysis.

2) \( A<> \) Controller.Finished
   allows to check, if the Controller reaches \textit{Finished} location in all evolutions. Verification shows that property holds.

3) \( E<> \) Controller.Finished and
   Controller.step == Controller.STEPS
   there exists such state, that the Controller finishes when all control commands were sent. The property holds for the model as well.

4) \( E<> \) Controller.Finished and
   Controller.step != Controller.STEPS
   checks, if all control steps were performed before reaching the final state of the Controller, i.e. property would hold if there exists at least one state in one path where Controller reaches \textit{Finished} state, but not all control commands were sent. It is formulated in such a way that when the tool returns negative answer, then the system works as expected. The property does not hold. It can be reformulated in a different manner
   \( A<> \) Controller.Finished and
   Controller.step == Controller.STEPS-1
   i.e. we can check, if in all paths eventually the state, where Controller has finished and it has made all steps is reached. It holds.

5) \( E<> \) HexaPOD.TargetReached and
   HexaPOD.current_pos == Controller.path[Controller.STEPS-1]
   there exists such state that HexaPOD reaches the target and its position coincides with the target position set by Controller.

6) \( E<> \) HexaPOD.TargetReached and
   HexaPOD.current_pos != Controller.path[Controller.STEPS-1]
   there exists such state that HexaPOD reaches the target and its position does not coincide with the target position set by Controller. Again, the property is formulated in such a way, that when the tool returns negative answer, then the system is corrected. As expected, the property does not hold.

Again, it can be reformulated in the following manner

\( A<> \) HexaPOD.TargetReached and
   HexaPOD.current_pos == Controller.
   path[Controller.STEPS-1].pos
i.e. we check, if in all paths eventually the state, where HexaPOD has reached target, and it coincides with the last target set by Controller.

Provided properties allow checking different characteristics of the systems and producing diverse diagnostic traces. The traces can be compared to the required trajectories, and the control properties of the HexaPOD as well as the Controller, estimated. More properties can be added. Moreover, traces can be exported and difference between the target and HexaPOD position calculated.

However, as it was already mentioned in [28], [29], it is easy to see that an average distance between the position of HexaPOD and its target should be found, and therefore exact durations are needed, and the change of the distance over time. Moreover, more realistic respiratory movement input are necessary. Therefore, hybrid models is required to estimate all durations and time scales.

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VI. Conclusions and Future Plans

A work in progress, the model of the radiation treatment system in Uppaal, is discussed. It is an abstract model, that includes selected elements of the complete system. It allows to obtain some useful characteristics of the system. Moreover, it shows certain limitations of the approach, time scales and corresponding distances (grid) should be chosen to get accurate results. In addition, current system does not allow providing realistic respiratory movement input, it is impossible to calculate average exposure of the healthy tissue.

Our conclusion is that such model is insufficient to answer all the interesting questions, and therefore it should be combined with hybrid model to get more information about the behavior of the system. Our future plans are as follows:

- Extensions of the Uppaal model:
  - model of HexaPOD with acceleration;
  - model of the targeting component;
  - implementation of the different control approaches.

- Continuous model of the HexaPOD, that would allow to build more exact discrete model, or generate discrete paths for timed model.

- Semi-formal control model in OpenModelica [30] (see [29] for the first attempt).

- Combination of the real respiratory movement trajectories and (formal) model to investigate systems adequacy to compensate it.

Moreover, hybrid model results should be used to modify existing models, namely distances and timing. Hybrid and timed simulation results should be compared to validate both models, and different control strategies should be analyzed with models.

REFERENCES
