Modification of Capital Budgeting under Uncertainty

The root purpose is a slight, pragmatic modification of the traditional discounted cash flow model of real investment under uncertainty. It is motivated by the occasional inadequacy of the Net Present Value (NPV) analysis with risk. It is done in accordance with the prescribed approach to capital budgeting suggested by the well-known textbooks in corporate finance. The traditional approaches are reviewed shortly.

**Keywords:** Capital budgeting, NPV, risk-adjusted discount factor, certainty equivalence.

**Introduction**

The allocation of capital investment is one of the most important and essential corporate decisions. At the present time the company acquires a bunch of real assets and enters a nexus of contracts with its shareholders, creditors, employers, customers, suppliers, and other stakeholders. The company keeps and governs the assets in the future; sometimes the investment concerns a considerable amount of assets measures in physical units and pecuniary units as well, and sometimes the investment concerns a sizeable span of time measured in years, even decades. The risk is that the decision is wrong, and the allocation is mistake; that is, the allocated assets could be used more efficiently for other purposes. In another language one can say that a substantial investment ties a huge amount of money for a long period with a latent risk of a mistaken decision which wastes some or all the invested money; that is the return is not sufficiently high to regenerate the capital laid out or stated differently, the rate of return on the investment is lower than the cost of capital. Hence, it’s decisive for the corporation and societal welfare that the corporation makes economically sound, correct and efficient capital investment; in another language, it is of vital meaning, that the firms and companies, or the investors if you prefer,
possess the ability to conduct an adequate appraisal and therefore know off the adequate “evolutionary and decision tools” (Arnold, Hatzopoulos, 2000).

**Capital Budgeting and the Capital Allocation Process**

The technique to demark, review, compare, and select the most appreciated investment project is denoted *capital budgeting*, according to widely recognised textbook in corporate finance T. Copeland, J.F. Weston (1988); R. A. Brealey, S. C. Myers, F. Allen (2003, or newer versions), and J. Berk and P. DeMarzo (2011)\(^1\). It is the obligation of the financial manager to select capital investments, to choose real assets, which by firm's business operations generate reasonable cash flows to return to the investors. Obviously, the decisive goal of financial management is to add as much value to the shareholders as possible\(^2\). Therefore the textbooks and many scholars conclude that capital budgeting and estimation of the cost of capital are the core of corporate financial decisions; consequently the financial managers must be equipped with the most adequate and satisfactory tools. Apparently, gradually, the rapid development in financial theory reaches capital budgeting technique; ten years ago J. R. Graham, C. R. Harvey (2001) seems ready to conclude quite cautiously that to some extent this apparent development commenced to be adapted by corporate financial practise.

Capital budgeting is a more narrow notion than the *capital investment allocation process* which embraces whatever related to capital investment at the company level; that is, the company’s mission and strategic context, the planning procedures (implementation and control), the screening of investment possibilities, the articulation of the different alternatives, the tools to appraise and evaluate the suggested investment projects, and the method to select the optimal investment, and the capital investment decision. The textbooks spend most of their words on the investment *appraisal and selection* methods, while the other initiatives devoted to allocation of capital are usually entrusted to the courses in strategy, strategic management, managerial accounting, etc. The present paper is likewise mainly devoted to the question of *investment appraisal*, nevertheless it’s worthwhile to remember that the very decision is not isolated in some vacuum (what economists too often believe), but belongs to an infrastructure and a series of activities of the corporation. J. L. Bower (1970) is a very early attempt to understand the corporation's long-term capital expenditure decisions in a much broader organisational and strategic context\(^3\).

**Aim and Attempt**

The financial theory has developed quite rapidly during the last 20 – 30 – 40 years both in relation to understanding, models, and tools and simultaneously textbooks in Corporate Finance has gone through adjustments and adaptions of new understandings, just compare R. A. Brealey and S. C. Myers from the first edition to the 10\(^{th}\) from 2010, or more dramatically, compare textbooks used in the fifties with the textbooks the students are provided today. However, it is not unfair to claim that the courses and textbooks still give pride of place to the well-known capital budgeting technique based on the discounting of the free cash flow with a suitable cost of capital. It is reasonable to expect that MBA and EMBA also in the future will teach financial managers how to use the well-known...
DCF (discounted cash flow) models in discrete time with uncertain cash flows, but without any explicit stochastic structure. Indubitably, attention will be drawn to the real option value of a cash flow, nevertheless my guess is – and it can easily be wrong – that future courses still will spend more time on the estimation of the risk-adjusted discount factor by means of CAPM (capital asset pricing model) than on the estimation of the option value of an investment project.

The present paper is not far from the traditional DCF model and accepts the pedagogical advantages of the model, simply the need to practise discounting in the appraisal of an investment project, but a slight modification is introduced. Opposite to the traditional teaching a certainty equivalent approach is used instead of the more cumbersome approach with the risk-adjusted cost of capital, A. Robichek, S. C. Myers (1966) for an early discussion. Although the purpose of the certainty equivalent approach in a multi-period setting of present value appraisal is to adjust for risk through running revision of the anticipated cash flows, then the approach seemingly apply an option of flexibility (Trigeorgis, 1996). The introduced modification is very simple, but the hope is that exactly the intended simplicity makes it imaginable to apply other option aspects.

The first attempt to look at capital budgeting problems as the present paper aims to do was S. C. Myers, S. M. Turnbull (1977). They wanted with the sure model in hand to assess the validity of more unreliable valuation methods like discounting by a risk-adjusted cost of capital. Osten-sibly, S. C. Myers, S. M. Turnbull, do not presume any explicit stochastic structure on the cash flow, just a random walk. In similar attempts by S. Bhattachary (1978) and C. Giaccotto (2007) they are in favour of a more explicit structure; they cannot conceive the potential prospect that cash flow of the capital budget wander away from a long-term mean, and both suggest a mean-reversing cash flow process. Except for the formal restrictions on the cash flow’s movements, it is not evident (in my eyes) what they achieve. G. A. Sick (1984) does not lay restrictions on the cash flow and use a free in-deterministic process of the cash flow – the present approach is mainly inspired by G. A. Sick.

The root purpose of this paper is the slight and pragmatic (as well) modification of the traditional DCF model of NPV analyses of an uncertain future cash flow as it is known from textbooks in corporate finance among others those mentioned above. The model is challenged at several fronts, more or less simultaneously: its relevance and practicability altogether in real life of strategic management. Moreover the paper sees it as an “assignment” to surround some of the problems with the conventional capital budget and its valuation.

The Net Present Value (NPV)

Apparently, Irving Fischer is one of the first economists who conceptualised NPV and applied the concept to a welfare analysis of the economy, when it seems that he reached the proposition that under the assumption of perfect markets, then NPV and the maximisation of NPV is on the road to optimal resource allocation of the economy, and moreover with Fischer Separation Theorem (which anticipates Modigliani and Miller) he states that on the efficient financial market the maximisation of NPV is equivalent to maximisation of the value of the firm and equity
value (the essential concordance between incentives). Unquestionably, the Fischer Separation Theorem is a milestone in economics and J. Dean with his book *Capital Budgeting* from 1951 presents the currently applied DCF methodology in NPV analyses.

Equivalent views are presented in the normative suggestions of well-known and well-reputed standard textbooks in corporate finance as e. g. the referred J. Berk and P. DeMarzo (2011), and R. A. Brealey, S. C. Meyers, and F. Allen (2010). Moreover, the two books are updated about capital budgeting and include modernised topics like e. g. binomial lattice and real option.

**Valuation**

Under the assumption that the valuation is done at the present moment \( t = 0 \) then the usual DCF valuation formula can be expressed as

\[
V_0 = \sum_{t=0}^{T} \frac{E\{ CF_t \}}{(1 + r_t + \kappa)^t},
\]

where \( V_0 \) is the present value of the stochastic cash flow stream, \( \bar{CF}_t \), and \( E\{ \bar{CF}_t \} \) is the expected value, the rate of discount is \( r_t + \kappa \), the spot riskless rate for loan, which matures in \( t \) periods, plus the constant risk premium. The tilde on top of a variable symbolises it as an uncertain variable.

When the valuation concerns an investment project, then the project’s NPV is \( V_0 \) deducted for cost of the project or the opportunity cost of the alternative project. The investment project is not rejected when the NPV is not negative. It is convenient to presume that the term structure of interest rates is constant; that is, \( r_t = r \) for all periods.

It is conventional in the financial textbooks to apply a single-period discount factor, based on CAPM. Hence, apply CAPM to anticipate what the required return on asset \( j \) in period \( t \) is

\[
E\{ \bar{R}_{j,t} \} = r_t + \beta_{j,t} \cdot [E\{ \bar{R}_{M,t} \} - r_t].
\]

where \( E\{ \cdot \} \) is an expectation operator, and the usual variables \( \bar{R}_{j,t}, \bar{R}_{M,t}, \beta_{j,t} \) are the return on asset \( j \), \( t \) – the return on the market portfolio of all the risky securities, and the systematic risk, beta of asset \( j \), respectively. The risk premium of asset \( j \) is

\[
\kappa_j = \beta_{j,t} \cdot [E\{ \bar{R}_{M,t} \} - r_t].
\]

One problem (even in single-period valuation) is that the asset’s systematic risk is determined by the covariance between the return on the asset and the market portfolio – a circulation. S. C. Myers and S. M. Turnbull (1977) note the further application of a single-period model’s discount factor with a constant risk-premium to a multi-period project is applicable only if systematic risk of project and market risk premium do not change over time.

The theoretical questions are under which conditions a constant risk-adjusted discount can apply in the discounting of the cash flow, and/or equivalently under which conditions the risk premium \( \kappa \) can claim to be constant. Unfortunately, it is not the convention in many textbooks to make precise when it is successful procedure. E. F. Fama (1977) makes precise the strict conditions for the procedure, (the theorem of M. J. Brennan (2003) do the same although in another set-up and an updated exposition): 1) the risk-free rate of interest and the market price of risk need in the multi-period to evolve deterministically over time, 2) the only admissible types of...
uncertainty are about the evolution of the expected value of the flow, 3) the risk adjustments in the discount rates must be known or non-stochastic at all relevant points in time. In conclusion if one is ready to accept the conditions “... in the case of an investment project of a given ‘type’, or in the case of a firm whose activities are not anticipated to change much in nature through time, it might make sense to assume that there is a single risk-adjusted discount rate or cost of capital that can be applied to all cash flows. Then the capital budgeting framework becomes much like that described in elementary textbooks”, see the exemplary explanation in L. Tri-georgis (1996, chapter 2) as well.

In the shortened interpretation cogitate

\[ V_0 = E \left\{ \sum_{t=0}^{\infty} \frac{CF_t}{(1+r_t+k)^t} \right\}. \]

After a short while the conclusion is either we need theory beyond the scope of the present paper, or it is acceptable that \( r_t, k \) are non-stochastic variables and their eventual changes are known prior. Hence, the only possible reassessment over the project’s lifetime is adjustment of the expected value of the cash flow the following period.

**Problems with the NPV Approach in Capital Budgeting**

The investment appraisal requires the ability of investor to specify the expected cash flow stream over the relevant periods, and moreover, it is required that the investor is able to determine a so-called hurdle rate, which ideally should be a discount factor with a properly adjusted risk premium. Formula (1) encounters more problems than the mere difficulties to maintain a single discount factor in a multi-period context. At least four problems can be stressed: A) the perception of risk, B) neglect of the option aspect in capital budgeting, C) the relative unimportance of the NPV criterion, and D) its “competitors” or rather supporters which are directed to section 3. Likely, the list is not depleted with those four points.

**A) Risk Perception**

Formula (1) rely on the application and the applicability of the rather unadventurous single-period CAPM. Furthermore, CAPM is theoretically subject to the mean-variance approach of the classical portfolio theory, only formally justified under the so-called elliptical distributions of return, S. A. Ross (1978), (with the normal distribution as a special case) or certain preference types as quadratic utility – not very attractive features. Empirically, the CAPM has been heavily attacked of E. F. Fama – French three factor model. E. F. Fama and K. French (1992) found in explanation of stock prices it seems that firm size and book to market value ratios are more reliable measures than the mere dependency on systematic risk. Finance provides for valuation and capital budgeting more general models like e. g. the use of a risk-neutral measure (an equivalent martingale measure). The variance is not attractive in the context of expected utility because it treats the same numerical deviation from the mean with the same weight, neglecting that risk-averse economic agents put more weight on negative deviations than positive deviations. The probability distribution of the future cash flow will be twisted in relation to skewness and kurtosis, E. F. Fama (1996), again with an inappropriate weight of downside risk.

It was stated at the commencement that the capital expenditures of a project are used to purchase a bunch of real assets
of different quality, types, economic life, etc. It is an important point that a multifarious and complex capital budget is a mix of different cash flow streams, each of them with different systematic risk, hinted e. g. by S. C. Myers and S. M. Turnbull (1977) and G. A. Sick (1984). It is far from straightforward how a single risk measure can account for multiple risk measures. Moreover, E. F. Fama and K. French (1992, 1993) and S. A. Ross (1976) indicate that a single cash flow stream generated from a single root can be impacted of many factors as well; even in simple cases it is not unusual that prices and revenue are more volatile than other budget items.

Although several practitioners follow the scholars’ recommendations and use sensitivity analysis, scenario and Monte Carlo analysis to get some idea about the range of the potential NPV and probability distribution of the NPV. Nevertheless, the impression is that the capital budget planning cares less about probabilities and more about the consistency of the balanced budget by means of a series of key ratios and key performance indicators based on historical accounting figures (if they exist), comparison with similar projects, or genuine analyses of relationships between the budget figures. Generally, capital budgeting procedures do the “unfailing” supposition of linearity and proportionality between the budget figures, which might cause unexpected deviations.

**B) Option Aspects**

The NPV criterion treats investment projects as “take it or leave”; if the project’s NPV is positive, then it should be implemented instantaneously, if not, then it should be trashed. Likely, the tactic is reasonable under circumstances without uncertainty, and the financial manager has no room for manoeuvre and no to response to forthcoming events during project implementation. Likely, such circumstances never occur. The simultaneous occurrence of risk and managerial flexibility, L. Trigeorgis (1996), is the frontrunner when it comes to valuable real options in capital budgeting and allocation of resources and capital expenditures. The sheer Creed is that flexibility has a value and adds value to the corporation; “willingness to adapt” and “adaptability” are nowadays the repeated mantras in the corridors of power and elsewhere in the company.

In consideration of capital budgeting it is customary to proclaim three genuine types of flexibility or options. Firstly, a budget can be planned with several contingencies which leave operational flexibility to response and revise earlier decisions. Secondly, the manager has an option of free entry and exit, the option to postpone the implementation of the project (the expected NPV increases by postponement) and the option to abandon ongoing projects (prohibits later losses) contingent on new information and resolution of assured uncertainties. Thirdly, manager has the option to adjust scale and scope.

The pivotal point is not the identification of the three types of flexibilities (it is not breaking news). Of course, the focus on real option forces managers’ awareness of embedded, valuable flexibility and the necessity to design a range of flexibilities – I believe, the pivotal point is the emerge and understanding of new models like Black-Scholes and the binomial lattice approach (both are now standard in the recognised textbooks), better tools and methodologies.
to handle stochastic processes evolving in time, and the rapid progress in financial econometrics.

Simply, the real option approach to capital budgeting affords better and more reliable estimate of the NPV when it is possible to come up with a price of the integrated option. Moreover, L. Trigeorgis (1996) shows than one more advantage to use option pricing to appraise project with integrated flexibility is to avoid the above-mentioned problem with the risk-adjusted rates discount factor in CAPM. Generally, L. Trigeorgis (1996) is valuable option.

C) Relative Unimportance of the NPV Criterion

The world demands great ideas, not boring calculations. It means to find the good long-term projects are a hard job, more difficult than capital budgeting. It is problematic to verify a probability distribution, if it exists, altogether; the markets are chaotic, and business events in real life appear similar to chocks, positive and negative, not in accordance with a conditional probability distribution and its conditional mean (Frank Knight “guesstimate”). Hence to manage the projects and response adequately to different non-estimable and non-quantifiable events and occurrence require certainly managerial flexibility, not in the sense of an estimable option price, but in the sense of open-mindedness and many other virtues. J. L. Bower (1970) showed early how organisational inertia requests good management practise in relation to external and internal team players; it is the power of verbal arguments, not budget figures, which commit the organisation to capital investment decision.

R. W. Adler (2006) reflects on the growing number of new topics, subjects, courses, etc. in a modern business school and management curriculum. Priorities are necessary; it is necessary to delete old courses to allow inclusion of new ones.

My first choice for elimination would be the use of discounted cash flow (DCF) for capital investment decision-making. The assumptions related to DCF are increasingly becoming so disconnected from business reality that its continued use should come with the following warning, ‘This financial management technique is hazardous to your business’; R. W. Adler (2006).

The main reason is that the investment calculations are too narrow-minded and offer too little significance compared to the hard facts in business with the unpredictable change of market conditions. It is more important with operational and strong mission and vision for the corporation – strategic management creates more value than sporadic calculations in financial management.

Competitors and Supporters

A List of Options

Corporate Finance assigns extensive and pretty wide-ranging efforts on the inputs like project appraisal to capital investment decision. The prevalent textbooks in corporate finance suggest a broad-spectrum of available approaches to capital budgeting. The suggestions can be the following options:

Sophisticated capital budgeting practices:

a) Monte Carlo Simulation
b) Real Options Method
c) Using Certainty Equivalents
d) Binomial Lattice
Advanced capital budgeting practices:

- Sensitivity analysis/Break-even analysis
- Scenario analysis
- Adaptation of Hurdle Rates
- Net Present Value (NPV)
- Adjusted Present Value (APV)
- Internal rate of return (IRR), modified internal rate of return (MIRR)
- Profitability index (PI)

Simple (naive) capital budgeting practices:

- Payback Period (PB), Discounted payback (DPB)
- Accounting Rate of Return (ARR)
- Earnings Multipliers and other equivalent multipliers.

The list is inspired by F. H. M. Verbeeten (2006). On top what he calls sophisticated capital budgeting practices (incorporate risk and flexibility options). In the middle the advanced capital budgeting practices (which consider time value of money, cash flows, and risk), and at the bottom the simple (or naive) capital budgeting practices (do not incorporate time value of money and risk systematically).

The traditional approach is the NPV rule which appreciates time value of money and accounts for the equity value – especially, in a world without risk and flexibilities; such a world is rather the exclusion than the imperative. Hence, it is understandable that the logic of the company pledges for alternatives and supplements to NPV, that the companies prefer an arsenal of investment criteria and different criteria to test the worth and value of a proposal than rely on a single criterion; in particularly if the company ponders over a sizeable and uncertain capital investment without an adjacent precedent or evidently correlated economic indicators.

A Few Surveys

Over the years several surveys studied the CFO’s (Chief Financial Officer) practise in capital budgeting by means of emitted questionnaires and personal interviews. In principle the different surveys rather archetypically query the corporations’ adoption of the approved financial analysis in the different textbooks, use of DCF, employment of formal risk analysis, post-auditing, suitable inflation adjustment, etc. An exception is e. g. J. R. Graham and R. Harvey (2001), which is a quite comprehensive research with a broader view and inclusion of more variables (independent as well as dependent ones) than the more “archetypical” interviews and questionnaires. Another example is F. H. M. Verbeeten (2006). Very often the purpose has been to investigate and inquire the so-called by G. Arnold, P. D. Hatzopoulos (2000) “theory-practice gap in capital budgeting”.

G. Arnold and P. D. Hatzopoulos observe in resemblance with other surveys that the sample of corporations over the years exhibits increasing sophistication in the practise of capital budgeting methods, but generally, not in the sense that more sophisticated methods substitute less sophisticated, rather the companies increase the arsenal of methods applied in investment appraisal (primitive, advanced and sophisticated methods go together, are complementary with each other, e. g. Pike (1996); Graham, Harvey (2001); Ryan, Ryan (2002); Verbeeten (2006); Block (2007)).

Sixty - forty years ago the surveys focused on the dissemination of cash flow methods compared with non-cash flow methods and the dissemination of discounted cash flow methods compared with methods not using cash flow and/or discounting,
D. F. Istvan (1961); E. F. Brigham (1975). Recent surveys\textsuperscript{18} document that DCF is accepted, together with non-cash-flow approaches and methods to handle risk and flexibilities in the capital budget; the companies exercise the option to use options in the capital budgeting\textsuperscript{19}.

\textbf{About the Gab}

It is not necessarily a surprise that the companies choose to enlarge the spectrum of approaches to evaluate capital investments, rather than renovate. Additionally, it is not necessarily a straightforward normative statement that the gab ought to diminish, rather, it seems more relevant for the companies to acquire a set of convenient and suitable methods; in any case, no real evidences substantiate the hypothesis that companies do have or had an outspoken resistance against new suggestions in finance – likely, it is a testable and tenable hypothesis that the companies try to keep up with the advances in corporate finance\textsuperscript{20}.

The more radical viewpoint is that it does not matter how the right decision is reached, it only matters it is the right decision. Less radical is the viewpoint that very often there is more method in the madness than offhanded believed. E. g. the earlier popularity of PB, IRR, PI in capital budgeting could be a smart reflection under conditions of less risk and more hard capital constraints: the three rather “naive” practises favour projects with liquidity and faster return of the invested capital more accurate than NPV. S. A. Ross (1996) mentions that asymmetric information with implied restriction on the available fund can explain the use of PB.

The efficient allocation of capital investment and the best use of capital expenditures are complicated matters. The corporation is confronted with substantial uncertainty, left to guesstimates and without sincere possibilities to quantify and verify the relevant multi-variable probability distribution: the decision-maker is forced to resort to proxies, and following R. Jagannathan and I. Meier (2002), R. L. McDonald (2000), and S. A. Ross (1996) quit often clever proxies. A representative quotation is R. L. McDonald (2000):

Most firms do not make explicit use of real option techniques in evaluating investments. Nevertheless, real option considerations can be a significant component of value, and firms which approximately take them into account should outperform firms which do not. This paper asks whether the use of seemingly arbitrary investment criteria, such as hurdle rates and profitability indexes, can proxy for the use of more sophisticated real options valuation. We find that for a variety of parameters, particular hurdle-rate and profitability index rules can provide close-to-optimal investment decisions. Thus, it may be that firms using seemingly arbitrary “rules of thumb” are approximating optimal decisions.\textsuperscript{21}

Of course ignorance can explain the application of bad and wrong methods in capital budgeting, but not for a long time due to competition from companies with adaptation of more efficient methods. However, we cannot refuse the incidence of bounded rationality in financial behaviour: the companies are not always perfect optimiser, but repeat workable routines, follow habits and use procedures which experientially reached satisfactory results, and sometimes in reality approximate nearly optimal procedures.

The next section presents a modification of discounted cash flow discounting.
Primarily, it is simple (transparent) way to modify for uncertainty in the anticipated cash flows.

**A Pragmatic Modification**

**CAPM, basically**

The proposed modification presumes that the single-period classical CAPM holds for the present moment and for each of all future periods. We do have as previously (2) what T. E. Copeland and J. F. Weston (1988) call the *return form* of CAPM.

\[
E\{\tilde{R}_{j,t}\} = r_t + \beta_{j,t} \cdot [E\{\tilde{R}_{M,t}\} - r_t],
\]

where \(E\{\cdot\}\) is an expectation operator, a tilde on top of a variable symbolises it is an uncertain variable, and else the well-known variables. By definitions

\[
\beta_{j,t} = \frac{\text{cov}(\tilde{R}_{j,t};\tilde{R}_{M,t})}{\sigma^2(\tilde{R}_{M,t})}, \quad \lambda_t = \frac{E\{\tilde{R}_{M,t}\} - r_t}{\sigma^2(\tilde{R}_{M,t})},
\]

where \(\sigma^2(\cdot), \text{cov}(\cdot, \cdot)\) is the variance operator and covariance operator, respectively. The exogenous parameter \(\lambda_t\) set by the market is called the *market price of risk*. Conventionally, it is assumed\(^{22}\) (AS.1) too *pragmatically* that both the risk free rate of interest and the market price of risk are constants over time; denoted \(r_t = r\) and \(\lambda_t = \lambda\) for all periods, \(= 1, 2, \ldots\). It is hardly crucial assumptions at the present, rather a notional matter of convenience and quite acceptable approximations, although theoretically we can’t refugue the variability\(^{23}\) and the modern flourishing financial econometrics can quite easily handle e. g. time-varying market price of risk, cf. for example M. W. Brandt and Q. W. Wang (2003) in another errand than the paper in hand.

Define the stochastic gross return of security

\[
j_{t+1} = \frac{\tilde{P}_{j,t+1} + \tilde{X}_{j,t+1}}{P_{j,t}},
\]

where \(\tilde{P}_{j,t+1}\) is the stochastic security price at the end of the period, and \(\tilde{X}_{j,t+1}\) is the stochastic cash flow paid to holder of the security at end of the period. After substitution of the new variables the expression is reformulated

\[
\frac{E\{\tilde{P}_{j,t+1} + \tilde{X}_{j,t+1}\}}{P_{j,t}} - 1 = r + \lambda \cdot \frac{\text{cov}(\tilde{P}_{j,t+1} + \tilde{X}_{j,t+1}; \tilde{R}_{M,t})}{P_{j,t}}.
\]


\[
P_t = \frac{1}{1+r} \cdot [E\{\tilde{P}_{t+1} + \tilde{X}_{t+1}\} - \lambda \cdot \text{cov}(\tilde{P}_{t+1} + \tilde{X}_{t+1}; \tilde{R}_{M,t})].
\]

The subscript to designate the security has been skipped to make the expression simpler. The numerator in (6) is the *certainty equivalent*. The security price is equal to an apparent certain cash flow discounted with the risk free rate of interest.

The single-period valuation model articulates the central recursive and dynamic relationship in valuation possible a bit more complicated than applied by finance students in basic valuation courses. It is solved recursively, backwards, and it is used subsequently to modified capital budgeting in a multi-period context, an early contribution is M. C. Bogue and R. Roll (1974).

**Capital Investment**

Previously, we thought at financial assets, now we change the view to real assets. The equivalent problem is to estimate
the present value of the future cash flow stream, which the company acquires through its acquirement of a bunch of real assets and establishment of a nexus of contracts in connection with a specific investment project or as part of the valuation of the entire corporation. The difference between the two kinds of estimation is considerable – some few of the more ostensible: a) the financial security is traded on a perfect, efficient market; the market for a specific asset can be pretty imperfect (maybe in particular for specialised assets), b) the return on a portfolio of securities is linearly combination of the individual returns; the cash flow of the combined assets is scarcely that (actually, it seems that lack of linearity is a value driver in capital allocation), c) the cost of regret is higher for real assets than financial assets, exit/entry from/to financial asset is more easier than that from/to a real asset, and d) likely, it's easier to estimate the statistical distribution of return on a bundle of financial securities than to estimate the statistical distribution of return on the bunch of real assets.

Keeping those remarks we mention the valuation model with a slightly changed notation,

\[ V_t = \frac{1}{1+r} \cdot [E_t(\bar{V}_{t+1} + \bar{CF}_{t+1}) - \lambda \cdot \text{cov}(\bar{V}_{t+1} + \bar{CF}_{t+1}; \bar{R}_{Mz})], \quad (7) \]

where \( V_t \) represents the value of the investment project at the commencement called date \( t \), and \( \bar{V}_{t+1} \) is the project's uncertain value at the end of the period called date \( t + 1 \), the stream of cash flows is symbolised by \( CF \). The value of the project at date \( 0 \) is \( V_0 \), and its cost at date \( 0 \) is \( I_0 \). Hence, the project is implementable date \( 0 \), if \( V_0 \geq I_0 \). Actually, no special reason for the change of notation, except for the subscript at the expectation operator, \( E_t(\cdot) \), which emphasises that expectation is taken contingent on all the available information at the beginning of the period.

The fiscal preparation and mounting of a capital budget for an investment project of some seize is a complicated affaire; many pieces of information from many different internal and external sources are merged together.

**Assumptions**

The planning date is the instant moment, date zero. At that date an investment project is defined by its expected cash flows,

\[ E_0(\bar{CF}_t) = \bar{CF}_t \quad \text{for} \quad t = 1, 2, ..., T, \]

where \( \bar{CF}_t \) is a stochastic variable of the cash flow at date \( t \), at date \( T \) the project is terminated, and \( E_s(\bar{CF}_t) \) is the expected value of the date \( tt \) cash flow at contingent on date \( s \) (\( t > s \)) and the available information at date \( s \). Furthermore, let \( cf_t \) denote the realised cash flow at date \( t \).

(AS.2) The mean of the true stochastic distribution of \( \bar{CF}_t \) is equal to planned budget item for date \( t \); that is, \( \bar{CF}_t \) is planned at date \( 0 \).

The crucial perception of the capital budgeting procedure is that the key item \( \bar{CF}_t \) is not really a probability distribution of \( \bar{CF}_t \). Through some standardised procedures the corporation reaches some estimates of the levels of the future cash flows,
and it is assumed quite frankly that the estimate of a cash flow is unbiased compared with the true mean value of the underlying probability distribution.

(AS.3) The cash flow \( \overline{CF}_t \) is a single flow, although more likely it is a mixed cash flow of several individual cash flows (different kind of revenues and cost items) such that each cash flow component has its own probability distribution and covariates separately with the return on the market portfolio.

Now we are going to make two strong (pragmatic) assumptions:

(AS.4a) \[ \overline{CF}_t = E_{t-1}(\overline{CF}_t) + \tilde{u}_t, \]

(AS.4b) \[ \text{cov}(\tilde{u}_t; \overline{R}_{M,t}) = \theta, \]

where \( \tilde{u}_t \) is a stochastic variable with mean value equal to zero and a finite variance. The first assumption (AS4.a) is a kind of random walk and says that the difference between the stochastic cash flow at date \( t \) and the expected value of the date \( t \) cash flow at the preceding date is stochastic. Next (AS4.b), the covariance between the mean deviation and the return on the market portfolio is constant during the time.

The second assumption is much stronger or “more pragmatic”:

(AS.5) \[ E_{t-1}(\overline{CF}_t) = \overline{CF}_t + \alpha \cdot [CF_{t-1} - E_{t-2}(\overline{CF}_{t-1})]. \]

The cash flow extrapolation at date \( t - 1 \) of the anticipated cash flow date \( t \) depends on two components; one is the originally estimate at date 0, \( \overline{CF}_t = E_0(\overline{CF}_t) \), the planner keeps the estimated level of the date \( t \) which is adjusted by the second component, a correction term, equal to the difference between the realised cash flow at the present instance and the anticipation at the previous date \( t - 2 \) of the date’s cash flow. Further, quite undramatically,

(AS.6) \[ \alpha \in (0, 1). \]

The Solution

The system of equations is solved recursively backwards. The procedure is hinted in the appendix. The solution is (A6) in the appendix.

\[
V_0 = \sum_{t=1}^{T} \frac{\overline{CF}_t}{(1+r)^t} - \lambda \cdot \theta \cdot (1 + \alpha) \cdot \frac{1-(1+r)^{-T}}{r} + \frac{\lambda \cdot \theta \cdot \alpha}{1+r}. \tag{8}
\]

The estimated value of the project becomes the present value of the originally planned budget items for the future cash flow discounted with the risk-free rate of interest. The expression accounts for the risk by subtraction of the present value of the (constant) amount of risk each date; that is, the product of the market price of risk \( \lambda \) and the covariance of the cash flow directly \( \theta \) plus its correction \( \theta \cdot \alpha \). The last a little inconvenient term is due to (AS.5) which works with a lag of one date. Alternatively, we do have

\[
V_0 = \sum_{t=1}^{T} \frac{\overline{CF}_t - \lambda \cdot \theta \cdot (1 + \alpha)}{(1+r)^t} + \left( \frac{\lambda \cdot \theta \cdot \alpha}{1+r} \right), \tag{9}
\]

each date is deducted a risk premium from budgeted cash flow. Both expressions lack slight intuitive power since it might be expected that the risk premium in seize is related to \( \overline{CF}_t \), however they gain in simplicity.

Seemingly, the two expressions function quite well as rules of thump with a translucent mode to allow for risk and
flexibility. If the wish is more attractive (natural) expressions, then it is necessary to modify the set of assumptions.

**Conclusions**

The traditional approach in capital budgeting under uncertainty suffers from several drawbacks, when a single-period risk-adjusted discount factor is used to estimate the present value of an uncertain multi-period cash flow. An alternative approach is the certainty equivalent approach which uses the risk-free discount factor. However, the certainty equivalent approach is not exonerated from the same drawbacks as it is based on the same philosophy (CAPM). Nevertheless the certainty equivalent approach is a more flexible approach, which we can utilise to build into the capital budgeting procedure a mechanism which accounts for the revision of the cash flows in the course of time. A simple model with few, transparent assumptions is applied. The belief is that the model due to its simplicity can overcome the resistance in practise to the certainty equivalent and can be developed further to incorporate real option analysis.

**Notes**

1. Especially the two last mentioned are designated for MBA and EMBA modules in corporate finance.
2. M. C. Jensen underlines clearly in the debate of shareholder value versus stakeholder value that the quest for economic welfare and economic efficiency corresponds with equity-value maximization (Jensen, 2001), and of course many more references exist.
3. Although J. L. Bower (1970) is quite early, it is still recommendable and readable. In my understanding it is one the first serious attempt to wrest financial management free from pure economics.
5. Both CAPM, Capital Asset Pricing Model, and APM, Arbitrage Pricing Model (Ross, 1976), are single-period models and encounter similar problems when applied to multi-period projects.
6. They argue that the assumption about constant systematic risk easily will be violated when a business or asset has growth potential, since the systematic risk of growth is likely to be higher than the systematic risk of investments already made and that this will cause the systematic risk of an asset to change over time. They argue that one approximation worth considering in this scenario is to change the risk-adjusted discount rate each period to reflect changes in the systematic risk.
7. The risk premium on the market portfolio divided with the variance of the market portfolio.
8. Provides a quite substantial characterization of the class of distributions. He shows that the CAPM can be derived from any two-fund separating distribution.
9. Well-known in the asset pricing of financial derivatives like Black-Scholes at efficient and complete financial markets.
10. Ernst and Young propose training programs in Russia, www.ey.com/cis/academy. Maybe a particular reference, but at least leaving doubt about the interest among practitioners and the more than willing consults surrounded them.
11. One sole exception (perhaps) is the employees who receive a redundancy notice in the hope for adaptability.
12. On a competitive market the firms could be trapped in a prisoner's dilemma because competition forces too early entry of the project implementation.
13. The determination of the optimal time to implement a project or close or project, or shift from one technology to another, etc. has for a long time been a topics in many courses in investment analysis – typical under certainty.
14. F. H. Knight in his book Risk, Uncertainty, and Profit from 1921. F. H. Knight distinguishes between three different types of probability, which are denoted: "a priori probability", "statistical probability" and "estimates". The third type appears when prediction and classification are excluded, only pure guesstimates are available.
Price – Earnings is a kind of payback evaluation.

The F. H. M. Verbeeten typology op. cit.

G. Truong et al. (2008) survey 356 firms listed on the ASX (Australia) August 2004. They report that nearly all respondents use NPV (94%) without suspend from payback (91%) and IRR (80%). They also found that value at risk is used (40%) and real option methods (32%). J. R. Graham and C. R. Harvey (2001) survey 392 CFOs US firms. Their results show that 76 % of the firm always or almost use IRR technique, and 1 % less to NPV method. P. A. Ryan and G. P. Ryan (2002) find that the most preferred technique is NPV (96 %), followed by IRR (92 %) and payback (75 %) from surveying Fortune 1000 firms receive with 205 responses.

J. R. Graham and C. R. Harvey (2001) report, that 27 % of firms in the sample incorporate real options or equivalent in project appraisal. P. A. Ryan and G. P. Ryan (2002) reach that 89 % of the companies never use real option methods and 79 % never apply option methods in capital budgeting. S. Block (2007) surveys CFO of Fortune 1,000 companies about their use of real options analysis as supplement to NPV criterion. 14 % of the respondents applied real options and 239 companies (85.7 % of them).

H.K. Baker et al. (2011) also survey CFOs of 214 Canadian companies to know how they think of real options technique in capital budgeting. However, real options method is least used; only 10.4 % of surveyed firms always or often use real options and 80.9 % of surveyed firms never used real options method in capital budgeting. G. C. Arnold and P. D. Hatzopoulos do not inquire the use of real option.

A quick test could to search on Google for courses in corporate finance and capital budgeting to financial managers.

The italic is mine.

We try to count the number of formal assumptions we do like AS.1, AS.2, etc. In reality, it might be a way to pull the wool over your eyes, because the hidden, implicit, and never outspoken assumptions could be much more controversially and textual obstacle.

The market price of risk varies to some extent in response to the time-varying of the aggregated risk-aversion which can be changing over the years with reciprocal effect on wealth.

References

The project terminates at date \( T \), hence the project value at this date is zero, \( V_T = 0 \), and we do have from (7),

\[
V_{T-1} = \frac{1}{1 + r} \cdot [E_{T-1}(CF_T) - \lambda \cdot \text{cov}(E_{T-1}(CF_T) + \bar{u}_T; R_{MT})].
\] (A1)

After substitution with (AS.4a)

\[
V_{T-1} = \frac{1}{1 + r} \cdot [E_{T-1}(CF_T) - \lambda \cdot \text{cov}(E_{T-1}(CF_T) + \bar{u}_T; R_{MT})]
\]

and utilise the covariance is linear, then
Apply (AS.5) to get

\[ V_{T-1} = \frac{1}{1+r} \cdot E_{T-1}(\bar{C}_{T-1} + \alpha) - \lambda \cdot \theta. \]

If the project terminates after one data then its present value is

\[ V_{T-1} = \frac{1}{1+r} \cdot \bar{C}_{T-1} + \alpha \cdot \left[ \text{cf}_{T-1} - E_{T-2}(\bar{C}_{T-2}) \right] - \lambda \cdot \theta. \]

At any date the present value of current cash flow looks like, (A2) and (A3).

If we split the project’s present value in two parts, one part discounts its value at the end of the period, the other part stems from its current cash flow, then at date \( T-2 \) then its value can be written

\[ V_{T-2} = \frac{1}{1+r} \cdot E_{T-2}(\bar{V}_{T-2} - \lambda \cdot \text{cov}(\bar{V}_{T-1}; \bar{R}_{M,T}) + E_{T-2}(\bar{C}_{T-1}) - \lambda \cdot \text{cov}(\bar{u}_{T-1}; \bar{R}_{M,T})]. \]

At date \( T-2 \) \( \text{cf}_{T-1} \) is a stochastic variable, and its expected value is \( E_{T-2}(\bar{C}_{T-1}) \), hence the correction term in (A3) vanishes. We get

\[ V_{T-2} = \frac{1}{1+r} \cdot E_{T-2}(\bar{V}_{T-2} - \lambda \cdot \text{cov}(\bar{V}_{T-1}; \bar{R}_{M,T})) = \frac{1}{1+r} \cdot E_{T-2}(\bar{C}_{T-1} - \lambda \cdot \theta - \lambda \cdot \theta \cdot \alpha). \]

If \( T \neq 2 \) then (AS.5) is substituted into \( E_{T-2}(\bar{C}_{T-1}) \), else we are finished. Adding (A4.1) and (A4.2) gives

\[ V_{T-2} = \frac{1}{1+r} \cdot E_{T-2}(\bar{C}_{T-1} - \lambda \cdot \theta - \alpha \cdot \lambda \cdot \theta) + \frac{1}{1+r} \cdot [E_{T-2}(\bar{C}_{T-1}) - \lambda \cdot \theta], \]

or after substitution with (AS.5) and reorganising,

\[ V_{T-2} = \frac{1}{1+r} \cdot [E_{T-2}(\bar{C}_{T-1}) - \lambda \cdot \theta - \alpha \cdot \lambda \cdot \theta] + \frac{1}{1+r} \cdot [E_{T-2}(\bar{C}_{T-1}) - \lambda \cdot \theta] + \frac{1}{1+r} \cdot \left[ \text{cf}_{T-1} - E_{T-2}(\bar{C}_{T-2}) \right]. \]

We continue repeating substitution with substitution with (AS.5), each time is added a term like \( \frac{1}{1+r} \cdot [E_{T-2}(\bar{C}_{T-1}) - \lambda \cdot \theta - \alpha \cdot \lambda \cdot \theta] \), except for \( t = 1 \), where the add will be \( \bar{C}_{T-1} - \lambda \cdot \theta \); since \( \text{cf}_0 \) does not exist and we pasted date \(-1\). The accomplishment is

\[ V_0 = \sum_{t=1}^{T} \frac{\bar{C}_{T-t}}{(1+r)^{t-1}} - \lambda \cdot \theta \cdot (1 + \alpha) \cdot \frac{1-(1+r)^{-T}}{r} + \frac{\lambda \cdot \theta \cdot \alpha}{1+r}. \]
Tačiau per paskutinius dešimtmečius NPV sulaukė nemažai rimtos kritikos (ir iš teoretikų ir iš praktikų). Apibendrinant galima paminėti keturias pagrindines kritikas: a) rizikos suvokimas (rizikos suvokimas vidutinės dispersijos sąlygomis yra retai realistinis); b) kitų kapitalo apskaitos galimybių neigimas (NPV kriterijus neigia keletą realių galimybų investiciniame projekte kaip, pavyzdžiui, projekto veiklos lankstumas ar galimybė atidėti ar atsisakyti projekto); c) santykinis NPV kriterijaus nesvarbumas (NPV yra paprastas ir ne tiek svarbus skaičiavimo atžvilgiu lyginant su atsirandanciomis komplikacijomis siekiant surasti strategiškai svarbius ir naudingus projektus); d) modelio konkurencinai (imones turi kitų priemonių, kurios, esant tam tikroms aplinkybėms, yra daug efektyvesnės nei NPV).

NPV kriterijus taikomas vieno periodo CAPM (liet. kapitolo įkainojimo modelis) modelyje įvertinti diskonto faktorui; keletas prieštaravimų egzistuoja šiam vertinimui, o tarp jų ir tai, kad ši procedūra neigia rizikos kitimą tam tikrame laikotarpyje. Tai gi, šiame darbe siūloma paprasta modifikacija, kaip patiksinti rizikos pritaikymą. Modifikacija turėtų būti pakankamai paprasta ir skaidri, kad galėtų būti pritaikyta praktikoje.