ON THE BIOGRAPHY AND THE SCIENTIFIC HERITAGE OF HERMANN MINKOWSKI

K. Pyragas and K. Svirskas
Department of Theoretical Physics and Informational Technology, Vilnius Pedagogical University, Studenty 39, Vilnius. LT-08106, Lithuania

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Abstract. Here we commemorate an outstanding mathematician and physicist, Hermann Minkowski, who was one of the developers of the Special Theory of Relativity. We discuss here some facts from Minkowski’s biography and his scientific heritage in physics.

Key words: history of astronomy – relativity – gravitation

1. INTRODUCTION

According to the decision of UNESCO and the World Union of Scientific Organizations the year 2005 is declared to be the “Year of Einstein” and the International Year of Physics. This special status of the year marks the century from the publication of Albert Einstein’s epoch-making works (Einstein 1905a,b,c), among them the fundamental work on the Special Theory of Relativity. This work has made a revolution in physics for the 20th century. This date obligates us to look at the development of physics at the crossroads of the 19th and 20th centuries and to commemorate the names of scientists who were at the origin of this revolution.

On 2004 June 22 we celebrated the 140th anniversary of the birthday of a great mathematician and physicist Hermann Minkowski. In this article we wish to remember his pioneering works which have substantially predetermined the development of physics up to the present time. For us this is an especially pleasant task since Hermann Minkowski was born in Aleksotas near Kaunas, Lithuania.

2. SOME BIOGRAPHICAL FRAGMENTS OF HERMANN MINKOWSKI

In this section we shall concentrate on some less known biographic facts of Hermann Minkowski, related to Lithuania. His comprehensive scientific biography is given in widely known farewell speech of David Hilbert (Hilbert 1909) and also in the book of Constance Reid devoted to Hilbert (Reid 1970). The biographical facts, till the autumn of 1872 when he and his family have moved from Aleksotas to Königsberg, are little known.

Hermann Minkowski was born in 1864 June 22 in Aleksotas (near Kaunas, Lithuania) in the family of a successful merchant. His father was of Jewish origin, his mother was German. Hermann was the third son in the family. His elder brother Mach did not receive a regular education, so he helped his father and later on inherited the business. The second brother Oscar was born in 1858 January 13.
His training began in elementary classes of the Kaunas provincial grammar school (now – the Maironis gymnasium). Though the place and date of birth of Hermann Minkowski are well-known in the world bibliography, but the information on the place of his birth till now is not sufficiently exact.

The above mentioned article of Hilbert (1909) is the most known source of Hermann Minkowski’s birthplace. It declares that “Hereman Minkowski wurde am 22 Juni 1864 zu Alexoten in Rusland geboren”. In the 19th century the Russian empire, frequently named Russia, included the extensive conquered territories belonging to various ethnic groups. As a consequence, the mentioning of “Alexoten”, a little-known suburb of Kaunas, has given a possibility to different interpretations and erroneous statements in the future. For example, Delone (1936) in his article, devoted to the mathematical works of H. Minkowski and based on the paper of Hilbert (1909), gives a strange modification of Hilbert’s words: “Hermann Minkowski was born on 1864 June 22 in a place of Alexot of the Minsk province...”. From this source the information almost literally has moved to the first edition of the Great Soviet Encyclopaedia (1938): “... was born in 1864 in a place of Aleksot of the Minsk province”. The same wrong information is repeated in the next editions of this Encyclopaedia (see references) and in other Soviet publications (e.g., Khramov 1977). However, Bogoliubov (1983) in his bibliographical publication is closer to the truth: “... was born in Aleksotas (now Lithuania, USSR)”. This discrepancy is distributed over many, especially Russian, sites in Internet.

Nevertheless, the birthplace of Hermann Minkowski “Alexot” is not some mystical place, but in those times it was already a sizeable settlement near Kaunas on the left coast of the river Nemunas, at its confluence with the Neris. The anterior name of the settlement (up to the 16th century) was Svirbigala, later it was named Aleksotas (or Alexotas). In 1795 Aleksotas together with Užnemunė has come to Prussia, in 1807 – to the Warsaw duchy and in 1815 has come to the Russian empire within the satellite Polish kingdom which existed up to 1830. Since 1918 Aleksotas belonged to independent Lithuania and in 1931 it was joined to Kaunas city.

At the time, when Minkowski family lived in it, Aleksotas was a part of Suwalki province with the Polish customs house in it and with operating laws different from Kaunas. This settlement is described in detail in the geographic directory published in Warsaw (Slovník geografický 1880). The directory writes: “Alexot is a village in the Maryjampol county, having 9250 of inhabitants... This is the place just on the bank of the Nieman river with a bridge, connecting it with Kowno. From the high hill, on the pedestal of which is Alexot, a wide-ranging view of Kowno opens... It is a brisk trade center, the vicinity grain market. In 1812 the Napoleon army crossed the Nieman river here... Alexot is also the hill on the left side of the Nieman bank, near the settlement with the same name. This hill was destined to goddess Alexote, who, together with Milda, were goddesses of love and right here was her temple.”

It is difficult to locate exactly the house in which H. Minkowski was born. There are only some indirect assumptions, not approved by authentic documents. Partially, in the abstract of lectures on physics of prof. Končius (1935), published by Kaunas University, is a note, that H. Minkowski “was born... in Aleksotas (the house against the bridge, now owned by A. Bronshtein)”. In the Kaunas archive one can find that a number of houses in Aleksotas were owned by A. Bronshtein. Only one of them, Angariečio 26, is still here, but it a question whether
H. Minkowski was born in it. The street, in which the Minkowski family lived, is known more definitely. The name of this street has been changed many times in the past. In 1922 the street was named “Minkauskio” (a Lithuanian version of the Minkowski family name). During the German and Soviet occupations the street get other names. However, during the M. Gorbachev “perestroika” period it became possible to return to the street its previous name. The street was named “H. and O. Minkovskiu”, in the honour of both brothers, Hermann and Oscar.

The information about Hermann’s childhood in Aleksotas is scarce (see Martišius & Pyragas 2004). Young Hermann sometimes was accompanying his father floating different goods (usually grain) along Nemunas from Aleksotas to Tilzit (in Lithuanian Tilžė), Memel (in Lithuanian Klaipėda) and Königsberg. Hermann was taught to read, write and calculate at home by his brother Oscar and his mother. Especially, he was skilful in mathematics and was able sometimes, together with Oscar, to help their father in bookkeeping. When Hermann was seven-years old, he was accepted to preparatory courses in the school which was on the right side of Nemunas river (in Kaunas), where his brother was already enrolled. Here his abilities soon were recognized, and he was transferred to the second year training group. In 1985 on the building of this school, which now is the Kaunas Maironis gymnasium, a memorial board was attached with the following text: “In this building a famous physicist and mathematician Hermann Minkowski attended preparatory classes of the foregone gymnasium in 1871–1872. The physiologist and doctor Oscar Minkowski attended the gymnasium in 1867–1872”.

Ethnographer Juozapavičius (1966) has recorded the witnessing of a teacher J. Kasiulaitis, that H. Minkowski, already being the famous scientist, visited his gymnasium in Kaunas in about 1905, where he was solemnly met and where he visited the most reminiscent places.

In 1872 (at this time Hermann was eight-year old) the family of Minkowski moved to Königsberg. The reasons for the departure from Aleksotas are not exactly known. Among them might be the prosecution of the father by Russian security services for his help to the participants of the 1863 uprising in Lithuania, the overall prosecutions of Jews in the imperial Russia, a flood of the Nemunas river, which caused large material damage to the business, etc. Reid (1970) describes this event by such words: “In 1872 the Prussian army triumphantly came back to Königsberg. But to David (Hilbert) more important was the fact that at that time the Jewish Minkowski family relocated to his city from a small town of Alexot near Kowno...”

Further biographic aspects of Hermann Minkowski are presented at length in
the cited papers (Hilbert 1909; Delone 1936; Reid 1970). In October of 1872 he joined one of the Königsberg gymnasia. Because of his excellent memory and insightfulness he graduated the gymnasium in a short time, being 15-year old.

Due to his extreme abilities even at this young age he was accepted at the Königsberg University to start studies in the spring of 1880. He studied five semesters at Königsberg, basically with professors Weber and Voigt, and three semesters in Berlin, where he listened to the lectures of Kummer, Kronecker, Weierstrass, Helmholtz and Kirchhoff. His mathematical capabilities were evident from the very beginning of the studies. As a result, in the first semester he received a monetary premium for the solution of one mathematical task. However, he refused this award for the benefit of the less solvent students.

Soon he started a deep and far-reaching research in mathematics. This research was related with solving the problem, declared by the Paris Academy of Sciences in spring of 1881. The task exposed on competition consisted in decomposition of the integer numbers into the sum of five squares. The 17-year old student perfectly solves this task, getting far out of the limits of the requirements. For this purpose he has developed a general theory of quadratic forms, especially the theory of their decomposition involving any indexes of inertia. It is a surprise that Minkowski at this age already was a master of algebraic methods, in particular, the theory of elementary dividers, and also of such transcendental methods, as Dirichlet series and the Gaussian sums. The paper titled with the motto “nothing is perfect except of a true, one true is fine” (“Rien n’est beau que le vrai, le vrai seul est aimable”) was handed to the Paris Academy on May 30, 1882. For this work in April of 1883 Minkowski was awarded the main prize “Grand Prix des Sciences Matematiques”.

H. Minkowski graduated the Berlin University in 1884. After that he worked as professor at the universities of Bonn (since 1887), Königsberg (since 1894), Zürich (since 1896), Göttingen (since 1902). In Zürich he was a teacher of mathematics of the other notability – Albert Einstein. Remembering his pupil, as the author of the well-known paper of 1905, Minkowski has noted: “Ah, that Einstein, he always passed the lectures – I would never believe, that he can make such things”.

All the works of H. Minkowski are collected in the compendium edited by Hilbert (1911) and described in his bibliographical article, devoted to Minkowski. This collection of papers is divided into four sections – 7 papers on the theory of quadratic forms, 14 papers on geometry of numbers, 6 papers on geometry and 5 papers on physics.

As a person, Hermann Minkowski was modest and friendly, and for these features his students and friends liked him. He was an example of diligence. Once he declared: “it is necessary to work, life is short”, and he worked till the last day of his life. His scientific work was interrupted by an attack of appendicitis, then an overdue operation and death on 1909 January 12.

Commemorating the 140th anniversary of the birthday of Hermann Minkowski let us remind, that his roots come from a very gifted family, which has given to the world the well-known doctor, physiologist and pathologist Oscar Minkowski (1858–1931), Hermann’s brother, and the astronomer Rudolf Minkowski (1895–1976), Oscar’s son.

3. HERMANN MINKOWSKI AND PHYSICS

Although H. Minkowski’s achievements in mathematics are great indeed, we believe that two papers published toward the end of his life in the field of theory

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of relativity (Minkowski 1908, 1909) have brought to him the greatest popularity in the scientific world. H. Minkowski’s interest to physics has appeared when he already was known as a mathematician. His first work in physics, submitted by H. L. F. Helmholtz, was on the motion of a solid body in a liquid (Minkowski 1888).

However, the most significant results of H. Minkowski in physics concern electrodynamics of moving bodies. The first work (Minkowski 1908) associates with the papers by Einstein (1905a) and Poincare (1906), devoted to the same branch of physics. The expression “Raum und der Zeit” used by Minkowski shows that at that time the space and time were not still considered as one, undivided manifold. However, we find here the rudiments of some ideas of his basic work (Minkowski 1909), because of the following, rather unambiguous attributes:

- the other term, Weltpunkt, is used for definition of the coordinates of the points of space and time; the “imaginary” time \( x_4 = i t \) and the designation

\[
x, y, z, t \Leftrightarrow x_1, x_2, x_3, x_4
\]  

(1)

are introduced;

- the four-vector electric current density is introduced:

\[
\rho v_x, \rho v_y, \rho v_z, \rho \Rightarrow \rho_1, \rho_2, \rho_3, \rho_4, \Leftrightarrow (j^\alpha = \rho v^\alpha, \alpha = 1, 2, 3, 4),
\]  

(2)

here \( v_x, v_y, v_z \) are the projections of the charge velocity and \( v^\alpha \) is its four-velocity;

- the four-dimensional tensor of the electromagnetic field (now – the tensor of Minkowski)

\[
F_{\alpha\beta} = \begin{pmatrix}
0 & H_z & -H_y & E_x \\
-H_z & 0 & H_x & E_y \\
H_y & -H_x & 0 & E_z \\
-E_x & -E_y & -E_z & 0
\end{pmatrix}
\]  

(3)

is introduced and its transformation rules under the Lorentz transformations are found:

- Maxwell’s equations are presented in the four-dimensional form:

\[
\Sigma_{\alpha=1}^4 \frac{\partial}{\partial x_\alpha} F_{\alpha\beta} = j_\beta;
\]  

(4)

- it is shown, that Lorentz transformations can be considered as hyperbolic (Lorentzian) rotations;

- it is shown, that the expression:

\[
ds^2 = dx^2 + dy^2 + dz^2 - dt^2
\]  

(5)

is invariant in Lorentz transformations (this was found earlier by H. Poincare, but without pointing out the consequences).

- the concept of a world line and the four-dimensional velocity as a derivative with respect to the proper time is introduced:

\[
u^\alpha = \frac{dx^\alpha}{d\tau}.
\]  

(6)
the concept of the four-force (Minkowskian force) is introduced, and the relativistic equations of the motion of a test particle are written:

$$m \frac{d}{d\tau} \frac{dx^a}{d\tau} = F^a;$$ (7)

- it is shown, that four-force and four-velocity vectors satisfy the following identities:

$$\left( \frac{dx}{d\tau} \right)^2 + \left( \frac{dy}{d\tau} \right)^2 + \left( \frac{dz}{d\tau} \right)^2 - \left( \frac{dt}{d\tau} \right)^2 = -1; F^a u_a = 0;$$ (8)

- first steps were made presenting the Newtonian law of gravitation in the Lorentz invariant form. The order of the general relativity corrections for the Solar system are estimated.

Consequently, in this paper H. Minkowski gives all the basic concepts of the four-dimensional understanding of the special theory of relativity, in which the design of the future building is seen. Only one step is missing, the step leading from a guess to the confidence. This step was done in the next work (Minkowski 1909). A deep conviction in truth of his theory is seen already from the opening words of this work: “M.H.! Die Anschauungen über Raum und Zeit, die ich Ihnen entwickeln möchte, sind auf experimentell-physikalischem Boden erwachsen. Darin liegt ihre Starke. Ihre Tendenz ist eine radikale. Von Stund an sollen Raum für sich und Zeit für sich völlig zu Schatten herabsinken, und nur noch eine Art Union der beiden soll Selbständigkeit bewahren.”

H. Minkowski was the first who noticed, that equations of Newtonian mechanics are characterized by double invariancy: in the first, they obtain their familiar form when changing the origin of the coordinate system and, in the second, moving the whole system as a rigid body. The basic idea about association of space and time into the uniform manifold he proves saying, that nobody had not observed any place differently, than at some moment of time, and any time differently, than in some place. Thus, there appears the basic concept – the four-dimensional world space and time, points of which are events. The sequence of positions of a point at different moments of time forms a world line. However, this design yet is not the basic revolutionary idea of H. Minkowski – in modern textbooks on classical (Newtonian) mechanics this design is used frequently. Revolutionism of his ideas begins with the search of invariant structures (not dependent on Lorentz transformations) in the theory of relativity. As a mathematician, he knew about ideas of relativity of geometrical constructions in our three-dimensional space with Euclidean geometry, which inevitably followed from Lorentzian transformations and the theory of relativity of Einstein (1905a). Namely, the rescue of geometrical idea as absolute (not dependent on the choice of inertial system of reference) he finds putting forward the two genial ideas:

- our real world is four-dimensional, uniting three-dimensional space and one-dimensional time (spacetime) and
- the geometrical structure (metrics) is a priori inherent only in the four-dimensional world and it is determined by the metric (Minkowskian metric)

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2,$$ (9)

that is invariant in Lorentz transformations.

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On the biography of Hermann Minkowski

Now the spacetime, characterized by the Minkowskian metric structure (9), is called the space of Minkowski. After that many conclusions in the Einstein theory of relativity become only geometrical character in the space of Minkowski. In this paper Minkowski formulates the generalized principle of relativity in the implicit form as an independence of physical laws from the choice of inertial systems of reference (their identity is established by Lorentzian transformations), and he calls this postulate the “world postulate”.

Not having the purpose to analyse all ideas in the given work, we shall specify only the most basic of them. So, having defined the spacetime metric structure (9), Minkowski for the first time introduces an indefinite metric which does not satisfy usual requirements of the metrics (the axiom of a triangle is not satisfied). He has found out some unusual properties of such space itself. In particular, it became clear to him, that in each point of the world line the spacetime invariantly breaks itself into three not crossing areas: the absolute future (Nachkegel), the absolute past (Vorkegel) and the light cone. This implies, that spacetime vectors will also be of three types, and Minkowski gives to them the names spacelike, timelike and isotropic, respectively. The condition of orthogonality, determined by metric (9) for such vectors is defined. The representation and analysis presented by H. Minkowski were so revolutionary, that Einstein has noticed (Sommerfeld 1949): “Since mathematics have grasped the theory of relativity, I do not understand it any more”.

The paper of Minkowski had a huge influence on the development of physics (and not only physics) in the 20th century. Undoubtedly, these ideas will be fruitful also in solving fundamental questions of physics in future. The first not trivial generalization of ideas of spacetime of Minkowski was carried out by Einstein (1916) in his successful idea to connect gravitation with the curvature of spacetime. The basic Einstein’s idea was, that gravitational field is responsible for the curvature of spacetime of Minkowski with specific geometrical structure. Thus, Einstein put forward a hypothesis, that at presence of gravitational interaction Minkowskian spacetime gets local structure of Riemannian space with the hyperbolic-type metric:

\[ ds^2 = g_{\alpha \beta} dx^\alpha dx^\beta. \]  

(10)

Here \( g_{\alpha \beta} \) is the metric of spacetime. These ideas have resulted in occurrence of the new experimentally established effects in the theory of gravitation: the redshift of spectral lines of atoms in gravitational field, the bending of light ray near the sources of gravitational field (gravitational lensing), the delay of light signals propagating in gravitational field (effect of Shapiro), the secular motion of the perihelion of planets (Mercury) in the field of the Sun. Large-scale investigations are now being conducted trying to detect propagation of small perturbations of gravitational field in the spacetime of Minkowski (gravitational waves).

The idea of spacetime formulated by Minkowski and Einstein has led to further, more radical generalizations. So, attempts are being made trying to generalize the idea of geometriztion to other types of interactions. These ideas became especially fruitful after introduction of the so-called procedure of compactification. This idea awards spacetime with a specific, inherent only to it, property of observable spacetime, while other dimensions are compactified in micro sizes of the order of the Planck size. These ideas now materialize in the theory of superstrings, supermembranes and other ideas in the Big Bang physics.

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Undoubtedly, that the idea of spacetime had a tremendous influence on the formation of modern understanding of the arrow of time. As it happens, the rate of growth of the arrow of time and the rate of growth of the arrow of space in the Universe (the expansion) are interconnected and counterbalanced by the Freedman’s (or De Sitter) law of expansion discovered by Hubble (1929).

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